London Equations

London eqns:

Near the surface, within a distance called the London penetration depth, the magnetic field is not completely cancelled. Each superconducting material has its own characteristic penetration depth.

Any perfect conductor will prevent any change to magnetic flux passing through its surface due to ordinary electromagnetic induction at zero resistance. The Meissner effect is distinct from this: when an ordinary conductor is cooled so that it makes the transition to a superconducting state in the presence of a constant applied magnetic field, the magnetic flux is expelled during the transition. This effect cannot be explained by infinite conductivity alone. Its explanation is more complex and was first given in the London equations by the brothers Fritz and Heinz London. It should thus be noted that the placement and subsequent levitation of a magnet above an already superconducting material does not demonstrate the Meissner effect, while an initially stationary magnet later being repelled by a superconductor as it is cooled through its critical temperature does.

\frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m}\mathbf{E}, \qquad \mathbf{\nabla}\times\mathbf{j}_s =-\frac{n_s e^2}{m c}\mathbf{B}. 

js: Superconducting current density

E: Electric field

B: Magnetic field

e: Charge on an electron and Proton

m: Electron mass

ns: phenomenological constant loosely associated with a number density of superconducting carriers

∇ X js: Curl js (infinitesimal rotation of a 3-D vector field)

\mathbf{j}_s =-\frac{n_se^2}{mc}\mathbf{A}. Both equations can be written as a single equation. Holds for mag-fields that vary slowly in space. **A** = Vector potential. Divergence of **A** is zero

Second eq­n manipulated by applying Ampere’s law\nabla^2 \mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}, \qquad \lambda \equiv \sqrt{\frac{m c^2}{4 \pi n_s e^2}}. \nabla \times \mathbf{B} = \frac{4 \pi \mathbf{j}}{c}

Notes:

London Equations mathematically describe the expulsion of a magnetic field from a material being cool to superconducting temperatures.

The magnetic flux equation can be manipulated with Ampère’s Law to obtain an equation that will determine the London Penetration Depth, which is where the magnetic field is not completely cancelled.